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Graphical Models

SYS 611: Systems Modeling and Simulation

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Agenda

1. Introduction to SysML/UML Diagrams
2. Dice Fighters Exercise

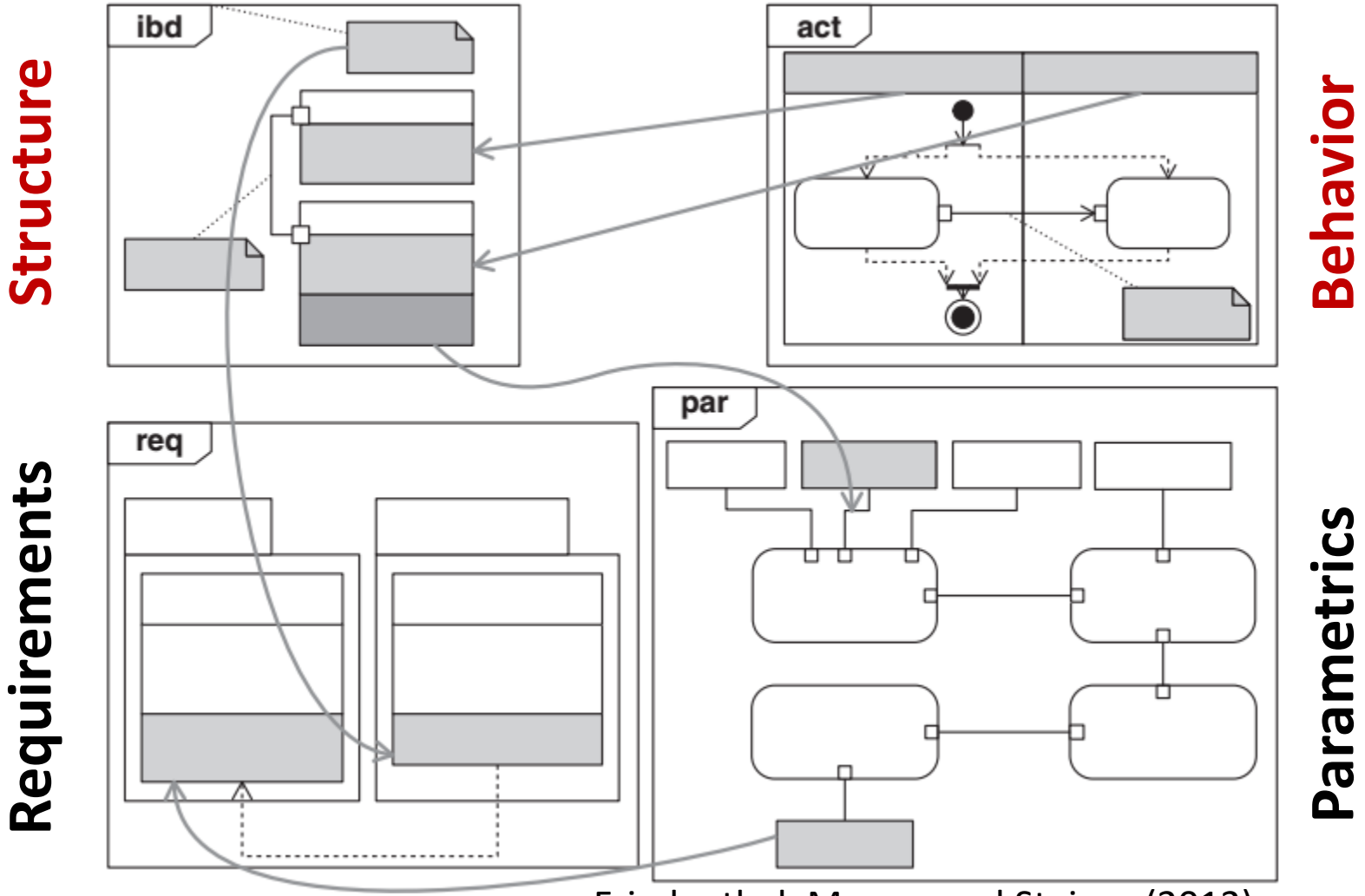
Reference: S. Friedenthal, A. Moore, and R. Steiner, "Getting Started with SysML," Ch. 3 in *A Practical Guide to SysML*, 3rd Edition, 2014.

Introduction to SysML/UML

Model-based SE (MBSE)

- MBSE transitions from document- to model-based information storage
  - Information is human- and computer-readable
  - Maintain models throughout analysis, specification, design, verification, and validation processes
- Models typically expressed in Systems Modeling Language (SysML) graphical language
  - Extension of Unified Modeling Language (UML)
  - See: Friedenthal, Moore, and Steiner, *A Practical Guide to SysML: The Systems Modeling Language*, 2014.

The Four Pillars of SysML



Friedenthal, Moore, and Steiner (2012)

Structure: Model State

The **model state** is the set of information required to recreate a snapshot in time

- Elementary state: minimal set of required information
- Derived state: additional information computed from elementary state (store only for convenience)
- Experimental frame is critical to identify state!

Image courtesy Steve Jurvetson (Wikimedia)

Behavior: State Changes

State changes represent logical transitions between model states

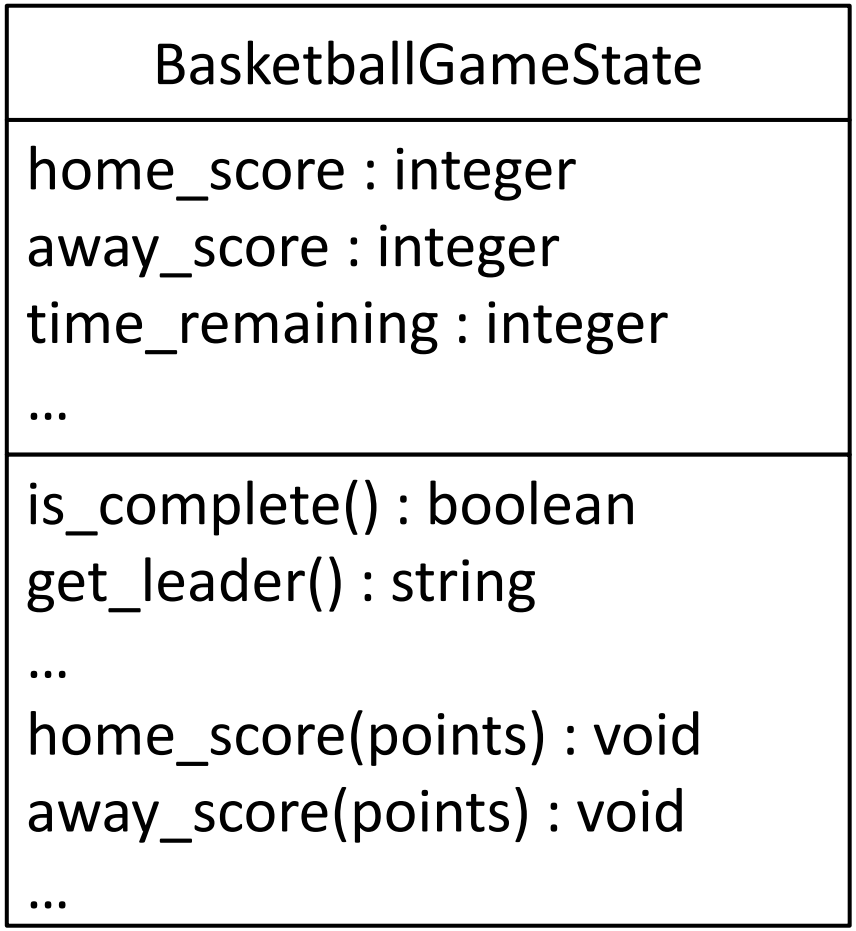
- Mathematically defined using “transition functions”
- Transform from: input state (with input arguments)
- Transform to: output state

Image courtesy Steve Jurvetson (Wikimedia)

Model State Diagrams

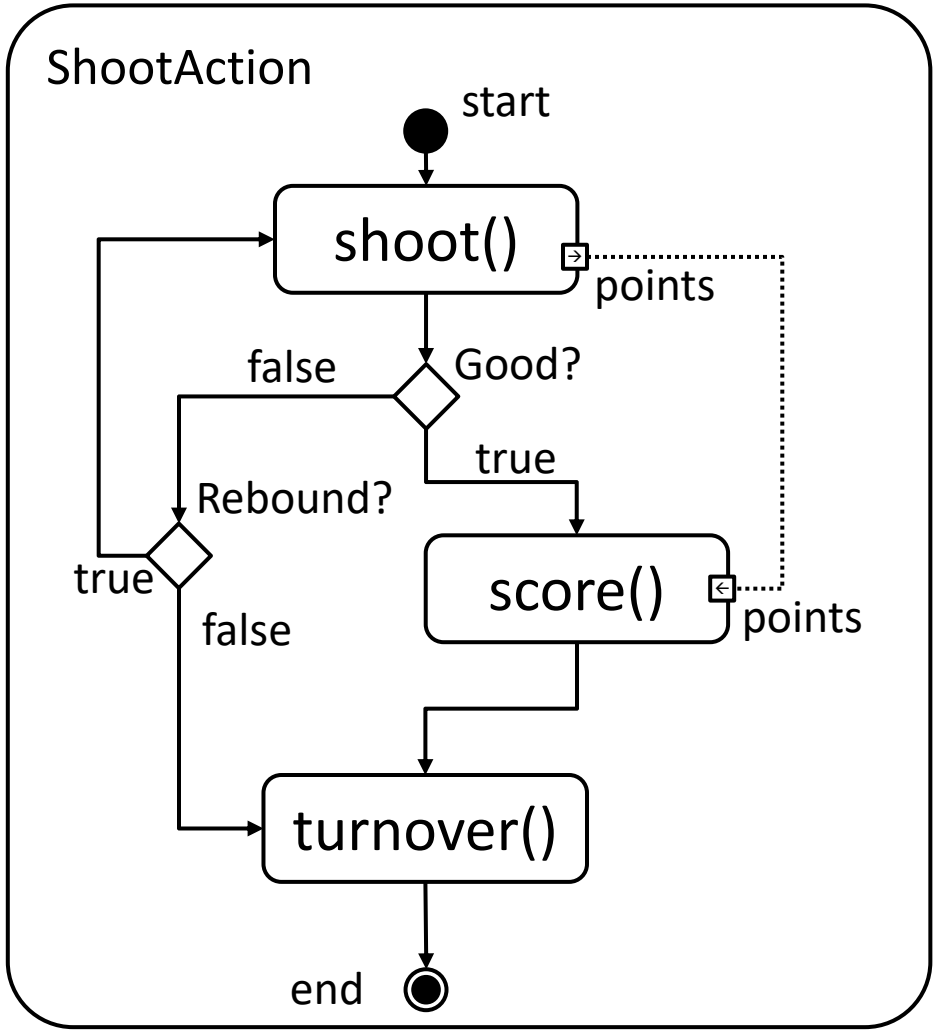
State diagrams illustrate the content and structure of model state information

- SysML block definition diagram (BDD)
- UML class diagram
- Diagram elements:
  - Elementary state
  - Derived state (function)
  - State transition function

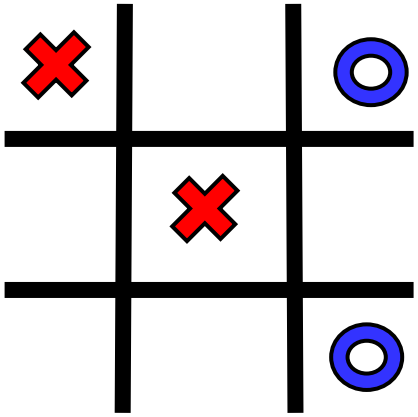


State Change Diagrams

- State change diagrams illustrate how to execute state transitions over time
- SysML/UML activity diagram (processes)
  - "Flow chart" perspective
  - Best at showing logical flow
- SysML/UML sequence diagram (interactions)
  - "Swim lane" perspective
  - Best at showing interactions



Example: Tic-Tac-Toe



Assume X always goes first.

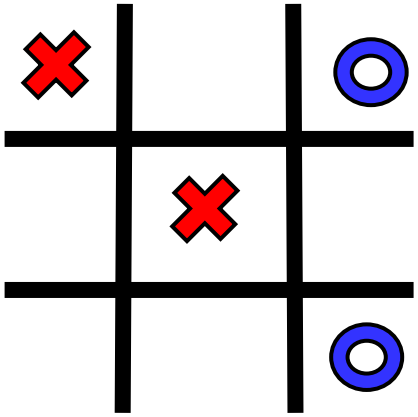
Model state

- What information is required to recreate a snapshot in time?
- What other derived state may be useful in Tic-Tac-Toe?

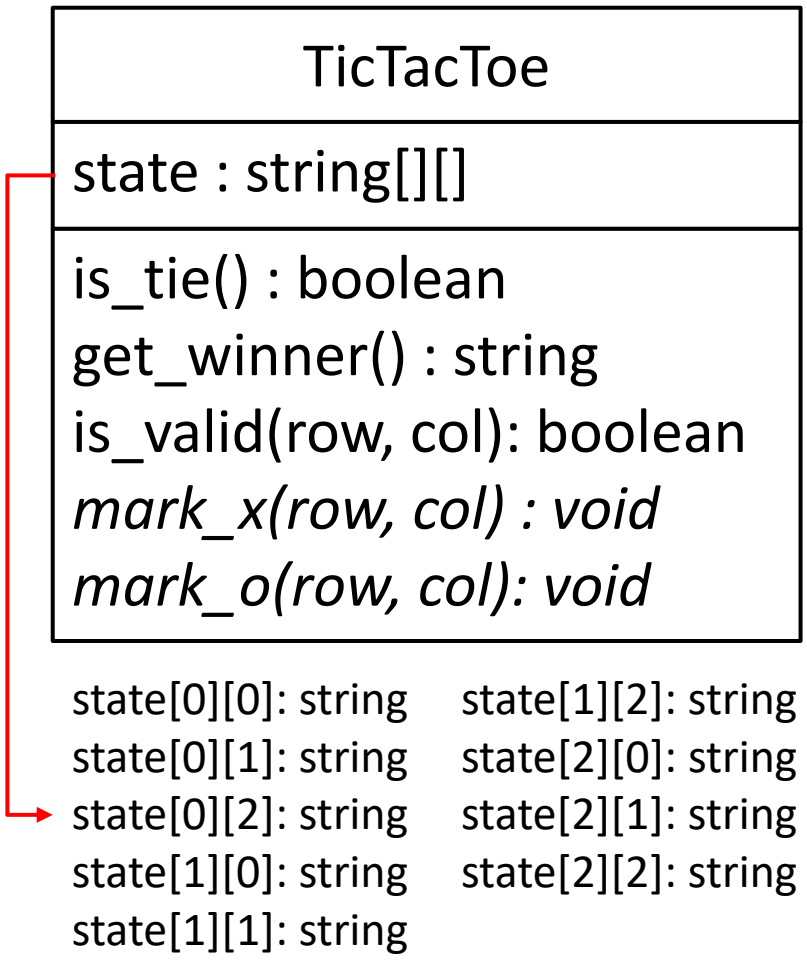
State changes

- What actions change the model state?

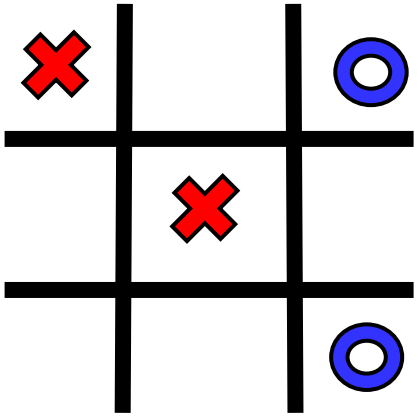
Tic-Tac-Toe State Diagram



Assume X always goes first.



Tic-Tac-Toe Activity Diagram



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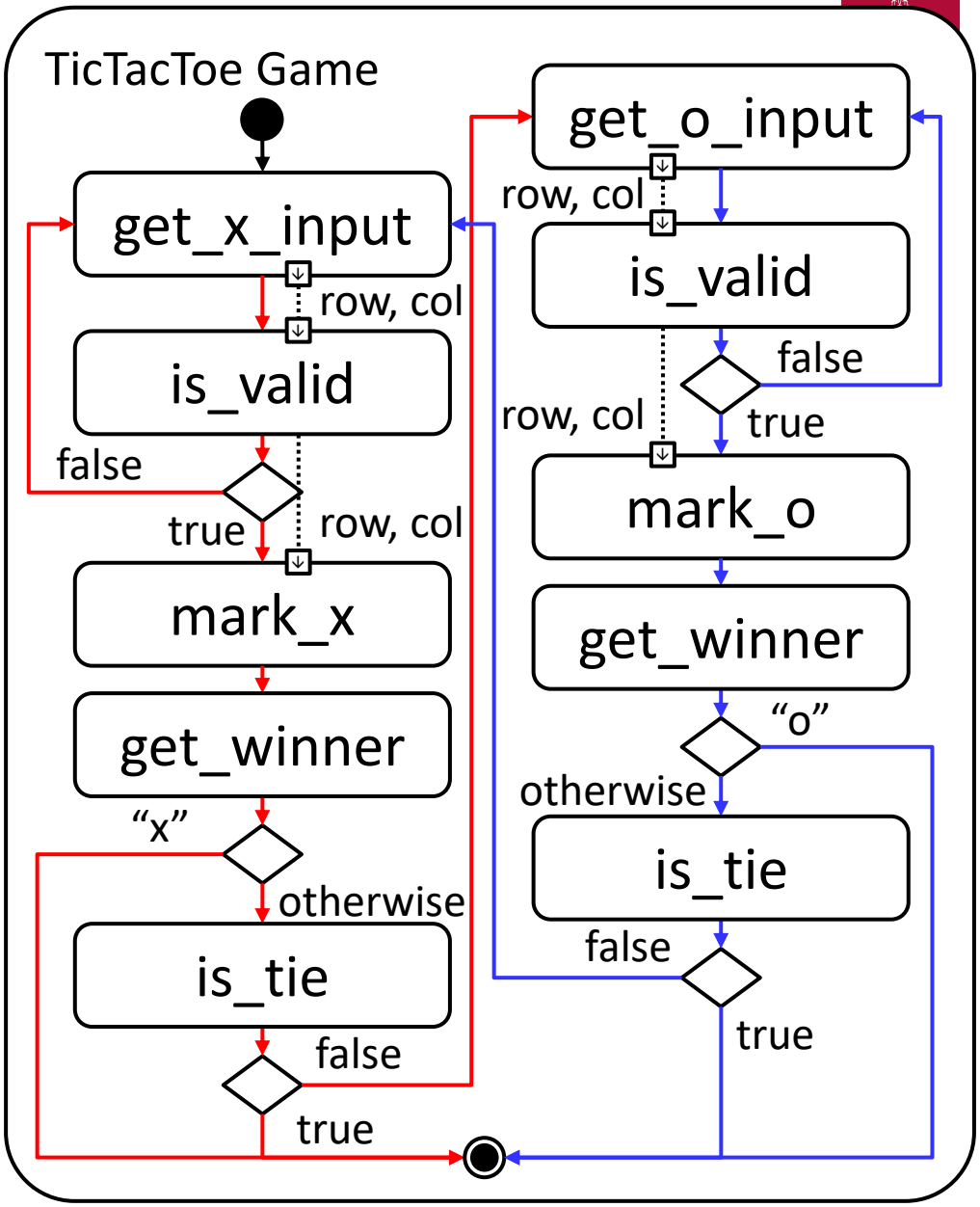
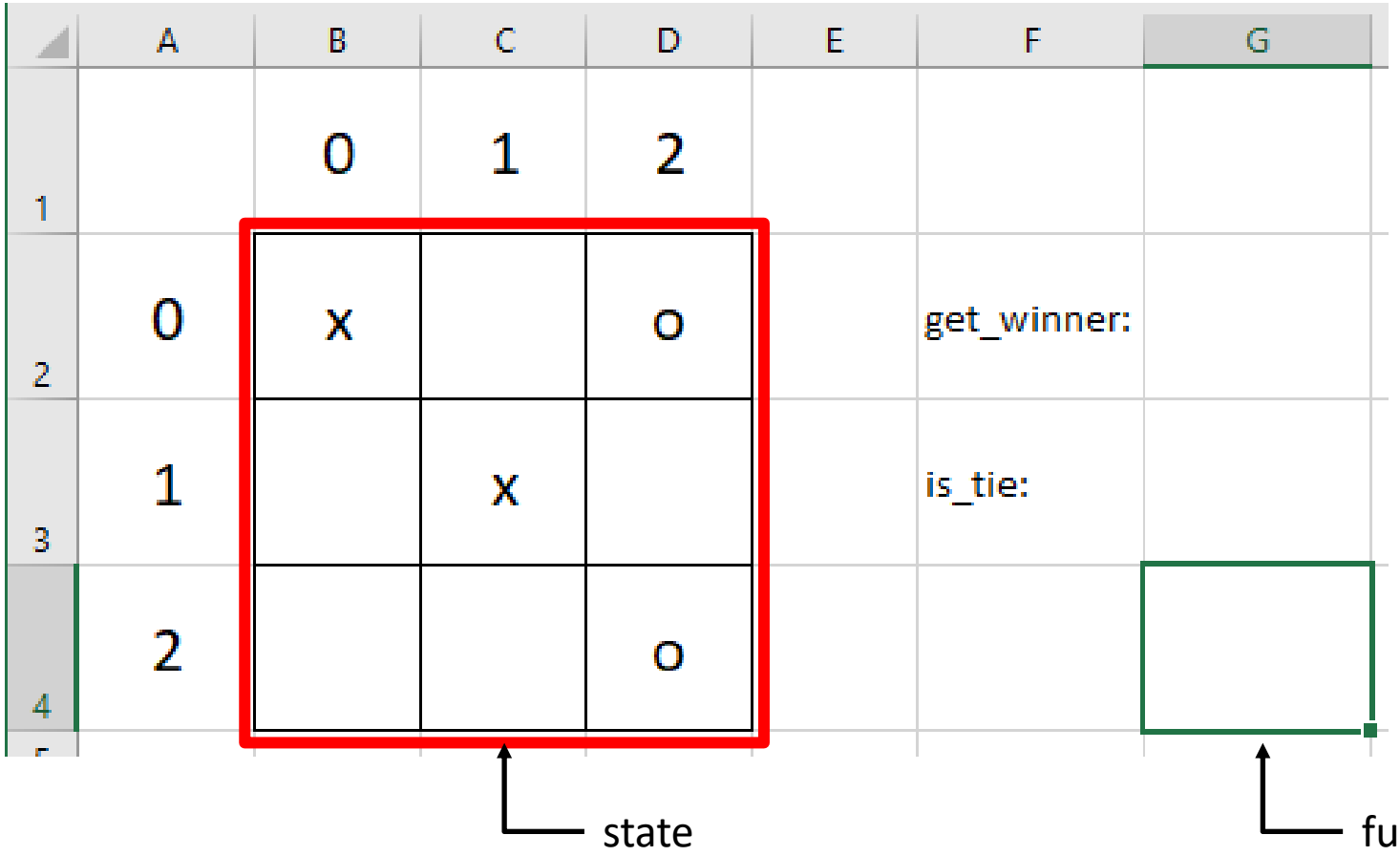


Tic-Tac-Toe Excel Sheet



Tic-Tac-Toe Python Script

```
import pandas as pd

state = [
    [" ", " ", " "],
    [" ", " ", " "],
    [" ", " ", " "]
]

def is_valid(row, col):
    return state[row][col] == " "

def mark_x(row, col):
    if is_valid(row, col):
        state[row][col] = "x"

def mark_o(row, col):
    if is_valid(row, col):
        state[row][col] = "o"
```

```
def show_grid():
    print(pd.DataFrame(state))

mark_x(1, 1)
show_grid()

mark_o(0, 2)
show_grid()

mark_x(0, 0)
show_grid()

mark_o(2, 2)
show_grid()
```

Dice Fighters Activity

Intro. to Combat Modeling

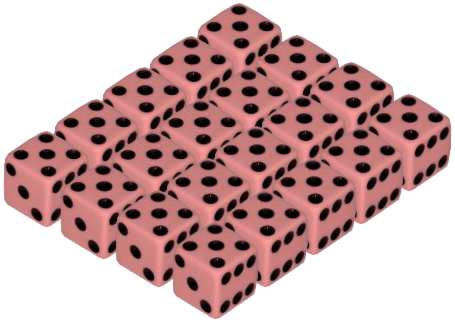
- Combat modeling simulates competing forces
- Long history of applications dating to 1800s

1:8000 Kriegsspiel (von Reisswitz, 1824) Photo by Roman März in P. von Hilgers (2000).

Dice Fighters Exercise

Red Team:

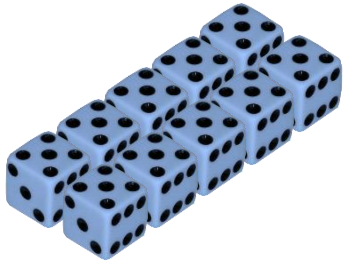
- 2x fighting force size
- Simple weapons



- Roll 6 to hit target

Blue Team:

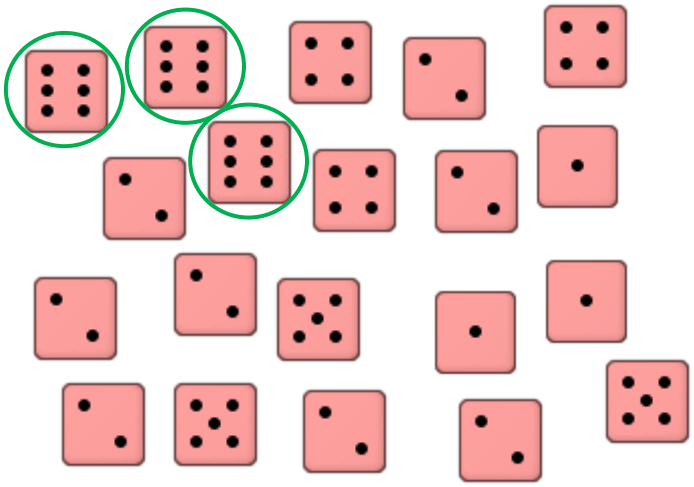
- Small fighting force
- 3x effective weapons



- Roll 4|5|6 to hit target

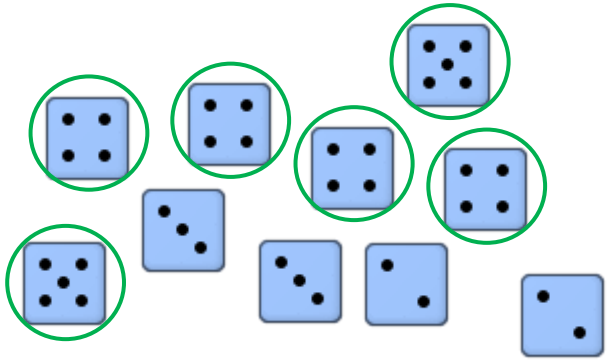
Dice Fighters: Example

- Roll 20 red dice



- 3 hits (≥6) – take away 3 blue dice for next round

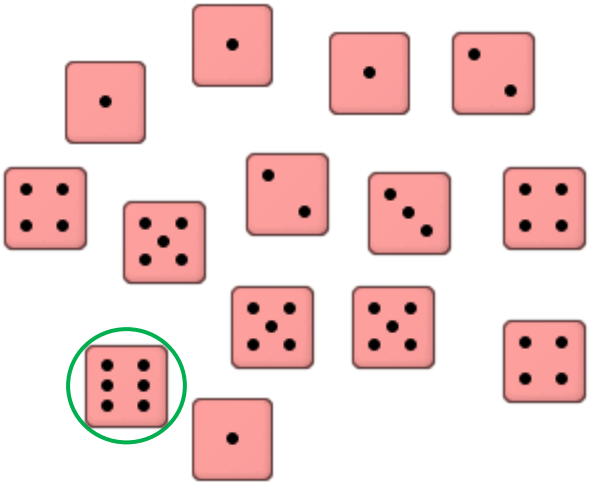
- Roll 10 blue dice



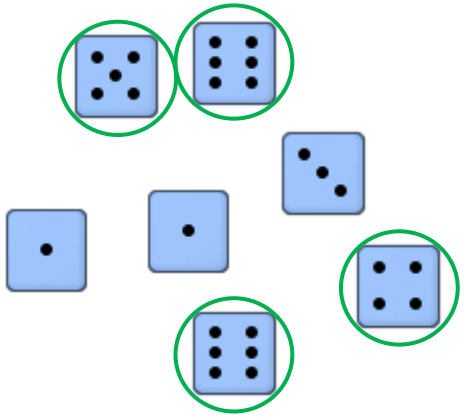
- 6 hits (≥4) – take away 6 red dice for next round

Dice Fighters: Example

- Roll $20 - 6 = 14$ red dice



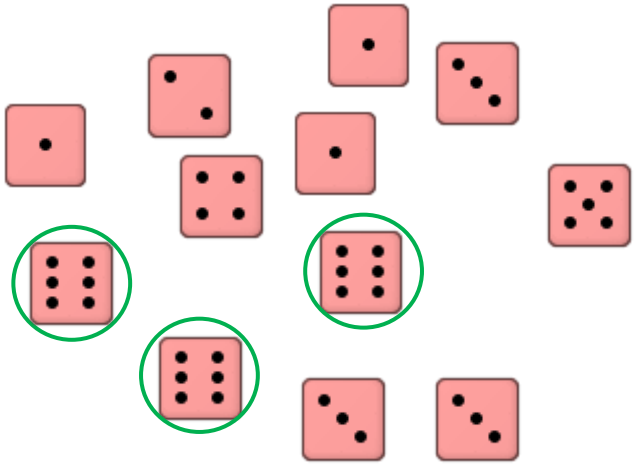
- Roll $10 - 3 = 7$ blue dice



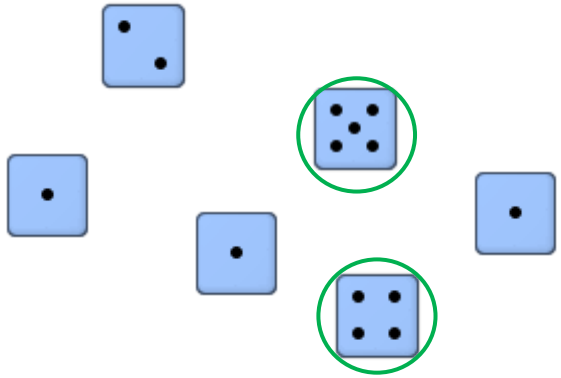
- 1 hits (≥6) – take away 1 blue dice for next round
- 4 hits (≥4) – take away 4 red dice for next round

Dice Fighters: Example

- Roll $14 - 4 = 12$ red dice



- Roll $7 - 1 = 6$ blue dice



- 3 hits (≥6) – take away 3 blue dice for next round
- 2 hits (≥4) – take away 2 red dice for next round

Repeat until one or both teams have no dice remaining!

Example State Trajectory

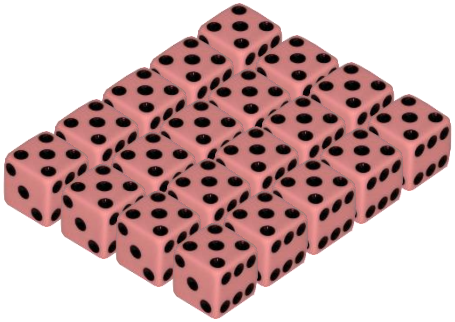
Round	Red Dice	Red Hits	Blue Dice	Blue Hits
0	**20**	3	**10**	6
1	20-6=**14**	1	10-3=**7**	4
2	14-4=**10**	1	7-1=**6**	3
3	10-3=**7**	2	6-1=**5**	4
4	7-4=**3**	0	5-2=**3**	3
5	3-3=**0**		3-0=**3**	

Blue is victorious in this sample

Dice Fighters Prediction

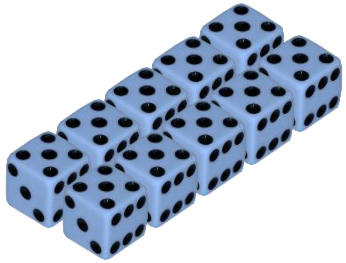
Red Team:

- Initial size of 20
- Roll 6 to hit target



Blue Team:

- Initial size of 10
- Roll 4|5|6 to hit target



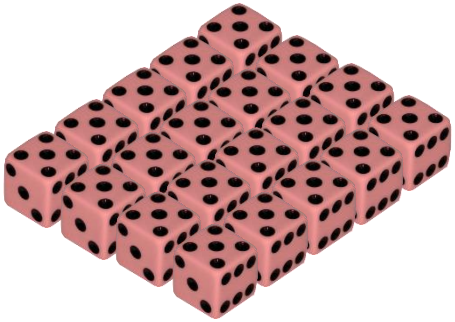
Which team has a stronger force?

pollev.com/pgrogan

Dice Fighters Experiment

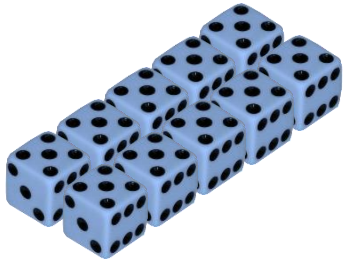
Red Team:

- Initial size of 20
- Roll 6 to hit target



Blue Team:

- Initial size of 10
- Roll 4|5|6 to hit target



Roll virtual dice with random.org/dice/

Submit results: pollev.com/pgrogan

Analytical Model

Time-rate change of red team:

$$\frac{dA}{dt} = -\beta \cdot B$$

Time-rate change of blue team:

$$\frac{dB}{dt} = -\alpha \cdot A$$

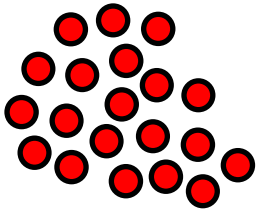
Solve initial value problem:

$$\frac{dA}{dB} = \frac{\beta}{\alpha} \cdot \frac{B}{A} \Rightarrow \alpha \cdot A \cdot dA = \beta \cdot B \cdot dB$$

$$\alpha \cdot (A_0^2 - A^2) = \beta \cdot (B_0^2 - B^2)$$

Lanchester's Square Law (1916)

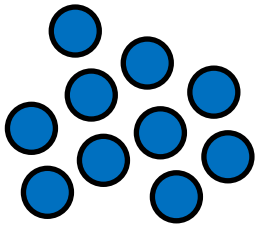
Red Team (A)



$A_0 = 20$

$\alpha = 1/6$

Blue Team (B)



$B_0 = 10$

$\beta = 3/6$

Analytical Solution

$$\alpha \cdot (A_0^2 - A^2) = \beta \cdot (B_0^2 - B^2)$$

Find required blue team "hit chance" for parity:

$$\beta = \alpha \cdot \frac{A_0^2 - A^2}{B_0^2 - B^2}$$

$$= \frac{1}{6} \cdot \frac{20^2 - 0^2}{10^2 - 0^2}$$

$$= \frac{1}{6} \cdot \frac{400}{100} = \frac{4}{6}$$

Classifying Dice Fighters

- Dice Fighters Experiment
  - System model > mathematical > simulation
  - Dynamic > discrete time
  - Stochastic
- Lanchester's Square Law
  - System model > mathematical > analytical
  - Static
  - Deterministic

Dice Fighters State Diagram

Model state

- What information is required to recreate a snapshot in time?
- What other derived state may be useful?

State changes

- What actions change the model state?

DiceFighters
round_number : int blue_size : int blue_chance_hit : float red_size : int red_chance_hit : float
is_complete() : boolean generate_blue_hits() : int generate_red_hits() : int blue_suffer_losses(int) : void red_suffer_losses(int) : void next_round() : void

Dice Fighters Activity Diagram

DiceFighters
round_number : int blue_size : int blue_chance_hit : float red_size : int red_chance_hit : float
is_complete() : boolean generate_blue_hits() : int generate_red_hits() : int blue_suffer_losses(int) : void red_suffer_losses(int) : void next_round() : void